A BALLISTIC MISSILE PRIMER

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The Rocket Equation

Consider a single-stage rocket with a lift-off mass M_{lo} and a burn-out mass M_{bo}. In the absence of gravity and air resistance, the change in the rocket's velocity from lift-off to burn-out (the "delta-v") is given by

$$\Delta v = v_e \ log_e\left(\frac{M_{lo}}{M_{bo}}\right) \tag{1}$$

where v_e is the exhaust velocity of the propellant. If the rocket has n stages, the total delta-v of the rocket is given by

$$\Delta v_t = \Delta v_1 + \Delta v_2 + \cdots + \Delta v_n = \sum_{i=1}^{n} v_{e_i} \ \log_e\left(\frac{M_{lo}}{M_{bo}}\right)_i \tag{2}$$

where v_{ei} is the exhaust velocity of the i^{th} stage and $(M_{lo}/M_{bo})_i$ is the ratio of the rocket mass when the i^{th} stage ignites to its mass when the stage burns out. The mass ratio of the i^{th} stage is given by

$$\left(\frac{M_{bo}}{M_{lo}}\right)_i = \frac{M_{t_1} + M_{t_2} + \ \cdots \ + M_{t_i} + m_p - M_{p_i}(1 - s_i)}{M_{t_1} + M_{t_2} + \ \cdots \ + M_{t_i} + m_p} = 1 - \frac{M_{p_i}(1 - s_i)}{\sum_{j=1}^{i} M_{t_j} + m_p} \tag{3}$$

where M_{p_i} is the propellant mass in the i^{th} stage, s_i is the fraction of propellant in the i^{th} stage that remains unburned, M_{t_i} is the total mass of the j^{th} stage before it is ignited, and m_p is the mass of the rocket payload. The unburned fraction is very low in modern boosters (e.g., $s = 0.0012$ in the Minuteman-II first stage) and can therefore usually be ignored. Thus, if we know the total mass, the propellant mass, and the exhaust velocity of each stage, we can estimate the total delta-v (and therefore the range) of the rocket for a given payload mass m_p.

Substituting equation 3 into equation 2, we find that the ratio of the payload mass to the total mass of the rocket at liftoff is given by

$$\frac{m_p}{\sum_{i=1}^{n} M_{t_i} + m_p} = \prod_{i=1}^{n}\left[1 - \frac{M_{t_i}}{M_{p_i}}\left[1 - \exp\left(-\frac{\Delta v_i}{v_{e_i}}\right)\right]\right] \tag{4}$$

Unfortunately, this equation cannot be solved for m_p as a function of Δv_t, because the Δv_i are functions of m_p. An approximation to equation 4 can be useful, however, for a quick estimate of the maximum payload capability of a missile. If we assume that the ratio of the initial stage mass to the mass of propellant burned is a constant f for all n stages [$(M_t/M_p)_i = f$], that the exhaust velocity of all stages is equal ($v_{e_i} = v_e$), and that the total delta-v is equally divided among the stages, ($\Delta v_i \approx \Delta v_t/n$), then

$$\frac{m_p}{M_{T'} + m_p} = \left[1 - f\left[1 - exp\left(-\frac{\Delta v_t}{n\, v_e}\right)\right]\right]^n \tag{5}$$

where $M_{T'}$ is the total mass of the rocket (without the payload). In the limit when the number of stages is large,

$$\frac{m_p}{M_{T'} + m_p} \cong \exp\left(-f\,\frac{\Delta v_t}{v_e}\right) \tag{6}$$

The rocket is a more efficient method of propulsion than is often assumed. Because virtually all of the chemical energy of the propellant is converted into kinetic energy of the exhaust, we can write the efficiency as follows:

$$\varepsilon = \frac{\frac{1}{2} m_p \Delta v_t^2}{\frac{1}{2}\sum_{i=1}^{n} M_{p_i} v_{e_i}} \tag{7}$$

If all of the stages have equal values of v_e, then

$$\varepsilon = \frac{\frac{1}{2} m_p \Delta v_t^2}{\frac{1}{2} v_e^2 \sum_{i=1}^{n} M_{p_i}} = \left(\frac{m_p}{M_P}\right)\left(\frac{\Delta v_t}{v_e}\right)^2 \tag{8}$$

where M_P is the total mass of propellant in all the stages. Assuming that $f_i = f$ for all stages, $M_{T'} = fM_P$. Solving equation 8 for m_p and substituting it into equation 5 and solving for ε, we have

$$\varepsilon = \frac{f\left(\Delta v_t / v_e\right)^2}{\left[1 - f\left(1 - \exp\left(-\frac{\Delta v_t}{n v_e}\right)\right)\right]^{-n} - 1} \tag{9}$$

If we assume, very roughly, that $\Delta v_t = n v_e$, then

$$\varepsilon = \frac{fn^2}{\left[1 - f\left(1 - \exp(-1)\right)\right]^{-n} - 1} = \frac{fn^2}{\left[1 - 0.63f\right]^{-n} - 1} \tag{10}$$

For example, if $f = 1.1$, $\varepsilon = 0.48$, 0.45, and 0.29 for $n = 1$, 2, and 3, respectively. In other words, a short-range, one-stage missile can be almost 50 percent efficient in converting the chemical energy of the propellant into kinetic energy of the payload, and even a long-range, three-stage missile can be almost 30 percent efficient.

The exhaust velocity is usually stated in terms of the specific impulse, or the impulse (force x time) produced per unit weight of propellant consumed. The specific impulse is related to the exhaust velocity by the equation

$$v_e = g\, I_{sp} \tag{11}$$

where g is the acceleration due to gravity at sea level (9.81 m/s^2). Note that the units of I_{sp} are seconds. The thrust is the exhaust velocity multiplied by the rate at which propellant is consumed:

$$T = v_e \frac{dM_p}{dt} = g\, I_{sp} \frac{dM_p}{dt} \tag{12}$$

If we assume that all the propellant is consumed during the burn time of the missile, t_{bo}, then the average thrust is given by

$$\hat{T} = g\, \hat{I}_{sp} \frac{M_p}{t_{bo}} \tag{13}$$

The specific impulse is a characteristic property of the propellant system, although its exact value varies to some extent with the operating conditions and design of the rocket engine (e.g., the combustion chamber pressure). The theoretical specific impulse for a variety of propellants is given in table 1. The specific impulse at higher altitudes is somewhat greater (due to the lower atmospheric pressure), so the average I_{sp} of a well-designed booster often exceeds the theoretical I_{sp} at sea level (especially for upper stages). Values of M_t, M_p, f

(M_t/M_p), $\hat{I}_{sp}$, and $\hat{T}$ for several U.S., Russian, and Chinese ballistic missiles are given in table 2.

Table 1. The theoretical specific impulse at sea level for a variety of rocket propellants.[a]

Propellant[b]	Examples	I_{sp} (s)
Liquid		
$H_2 + O_2$	Space Shuttle, Saturn	390
RP-1 + O_2	Thor, Atlas	300
NTO + N_2H_4/UDMH	Titan-II	288
NTO + UDMH	SS-17, SS-18, SS-19	285
IRFNA + UDMH	DF-3, Scud-B, Lance, Agena	275
EtOH + O_2	V-2	279
Solid		
AP/Al composite	Minuteman, MX, Trident	265

Source: George P. Sutton, ***Rocket Propulsion Elements*** (New York: John Wiley & Sons, 1986).
[a]Assuming a combustion chamber pressure of 1000 psi, a nozzle exit pressure of 14.7 psi, and an optimum nozzle expansion ratio.
[b]H_2 is liquid hydrogen, O_2 is liquid oxygen, RP-1 is kerosene, NTO (nitrogen tetroxide) is N_2O_4, N_2H_4 is hydrazine, and UDMH (unsymmetrical dimethylhydrazine) is $(CH_3)_2$NNH2. IRFNA (inhibited red fuming nitric acid) is about 85% nitric acid (HNO_3), 13-15% NO_2, and 0.5% HF. EtOH (ethyl alcohol) is C_2H_5OH. AP (ammonium perchlorate) is NH_4ClO_4; a polymer binder holds this together with aluminum (Al).

Note that the actual burn-out velocity of the rocket will be less than Δv_t if forces other than thrust, such as gravity and air resistance, act on the rocket during its burn. The actual burn-out velocity can be calculated if the characteristics of the rocket are known in considerable detail. For most purposes, however, it is convenient to assume that the burn-out velocity is simply the total delta-v minus a correction for air and gravity that is relatively insensitive to changes in payload:

$$v_{bo} = \Delta v_t - \Delta v_{ag} \tag{14}$$

In general, slower-burning rockets have larger values of Δv_{ag}, and first stages have a higher Δv_{ag} than later stages. Table 3 gives approximate values of Δv_{ag} for several types of missiles.

Table 2. Total booster mass M_t, propellant mass M_p, average specific impulse I_{sp}, average thrust T, burn time t_{bo}, and burn-out altitude h_{bo} for several ballistic missiles.

Missile	Stage	M_t (kg)	M_p (kg)	$\frac{M_t}{M_p}$	I_{sp} (s)	T (kN)	t_{bo} (s)	h_{bo} (km)
Minuteman II[a]	1	23,230	20,780	1.12	267	899	61	34
	2	7,270	6,230	1.17	284	266	65	96
	3	2,010	1,660	1.21	275	76	59	
Minuteman III	3	3,710	3,306	1.12	285	152	61	190
MX[b]	1	48,700	44,300	1.10	284	2315	53	22
	2	27,800	24,900	1.12	304	1365	54	82
	3	8,200	6,790	1.21	306	329	62	196
Pershing II[b]	1	4,110	3,580	1.15	276	172	58	18
	2	2,600	2,250	1.16	279	134	46	
Titan II[c]	1	116,850	112,720	1.04	288	2170	147	
	2	26,810	24,150	1.11	313	407	182	315
SS-18[d]	1	171,000	154,900	1.10	317	5670	85	
	2	38,500	36,000	1.07	337	992	120	
DF-3[e]	1	65,500	61,400	1.07	241	1020	140	100
V-2[f]	1	11,800	8,800	1.34	198	252	68	28
Scud-B[g]	1	4,900	3,700	1.32	240	120	70	30

[a]John Simpson, Aerojet General, Sacramento, CA, and Larry Hales, Thiokol Chemical Corporation, Brigham City, UT personal communication, 15 July 1991. Stages 1 and 2 of Minuteman III are identical to those of Minuteman II.
[b]"Short burn time ICBM characteristics and considerations," (Denver, CO: Martin Marietta, 20 July 1983).
[c]"Titan II Space Launch Vehicle: Payload Users Guide," (Denver, CO: Martin Marietta Corporation, August 1986).
[d] Rolf Engel, "The SS-18 Weapon System," ***Military Technology***, Vol. 13, No. 3 (March 1989), pp. 112-121.
[e]Zuwei Huang and Xinmin Ren, "Long March Launch Vehicle Family—Current Status and Future Development," ***Space Technology***, vol. 8, no. 4 (1988), pp. 371-375.
[f]Gregory P. Kennedy, ***Vengeance Weapon 2: The V-2 Guided Missile*** (Washington: Smithsonian Press, 1983).
[g]Steven Zaloga, "Ballistic Missiles in the Third World: SCUD and Beyond," ***International Defense Review***, Vol. 21 (November 1988), pp. 1423-1427.

Table 3. Approximate values of Δv_{ag} for solid and liquid stages of long-range missiles.

	Δv_{ag} (km/s)	
Stage	Solid-fuel	Liquid-fuel
1	0.6	1.1
2	0.15	0.6
3	0.15	---

Source: E.H. Sharkey, "The Rocket Performance Computer," RM-23003-RC (Santa Monica: The RAND Corporation, 1959).

A more precise method of calculating the burn-out velocity is to solve numerically the equations of motion of the missile. If the missile's thrust vector is aligned with its velocity vector (i.e., gravity is used to turn the missile), the equations of motion are:

$$\frac{dv_x}{dt} = \left[\frac{T - \frac{1}{2}\rho A C_d v^2}{M}\right]\frac{v_x}{v} - \frac{GM_e\, x}{\left(x^2 + y^2\right)^{3/2}} \tag{15a}$$

$$\frac{dv_y}{dt} = \left[\frac{T - \frac{1}{2}\rho A C_d v^2}{M}\right]\frac{v_y}{v} - \frac{GM_e\, y}{\left(x^2 + y^2\right)^{3/2}} \tag{15b}$$

where v_x and v_y are the components of the velocity vector v in the x and y directions, T is the thrust, ρ is the atmospheric density (a function of altitude), A is the cross-sectional area of the missile, C_d is the drag coefficient (a function of v), M is the missile mass (a function of time, as propellant is consumed and empty stages are jettisoned), G is the gravitational constant (6.67×10^{-20} km^{3}s^{-2}kg^{-1}), M_e is the mass of the Earth (5.97×10^{24} kg), and x and y are measured from the center of the Earth. The atmospheric density as a function of altitude is given in table 4; table 5 gives the drag coefficient of the V-2 missile as a function of velocity.

If the rate of propellant use is constant, all of the propellant is consumed during the burn, and the empty rocket body is jettisoned after burn-out, the mass is equal to

$$M(t) = M_t - M_p(t/t_{bo}) + m_p \qquad [t \le t_{bo}] \tag{16}$$

$$M(t) = m_p \qquad [t > t_{bo}]$$

where the $t_{bo} = g\, M_p \hat{I}_{sp} / \hat{T}$. This equation is easily generalized to n stages.

Table 4. Atmospheric density ρ and scale height H as a function of altitude.[a]

Altitude (km)	Density (kg/m^3)	Scale Height (km)	Altitude (km)	Density (kg/m^3)	Scale Height (km)
0	1.225	10.42	20	0.0889	7.62
1	1.112	10.30	30	0.0184	7.15
2	1.007	10.19	40	0.00400	6.99
3	0.909	10.06	50	0.00103	7.06
4	0.819	9.95	60	3.10×10^{-4}	7.24
5	0.736	9.82	80	1.85×10^{-5}	7.20
6	0.660	9.70	100	5.60×10^{-7}	6.85
8	0.526	9.46	150	2.08×10^{-9}	7.43
10	0.414	9.21	200	2.54×10^{-10}	8.97
15	0.195	8.16	300	1.92×10^{-11}	12.06

Source: National Oceanic and Atmospheric Administration, "U.S. Standard Atmosphere, 1976" (Washington, DC: NOAA, 1976).

[a]This table should be interpolated using the function $\rho(z) = \rho(0)\exp(-h/H)$, where H is the scale height (interpolated using the values given in the table).

Table 5. The drag coefficient C_d of the V-2 missile as a function of its velocity v.

Velocity (Mach)	C_d	Velocity (Mach)	C_d
0.0	0.25	2.5	0.15
0.5	0.18	3.0	0.14
1.0	0.28	3.5	0.12
1.2	0.36	4.0	0.11
1.5	0.26	5.0	0.10
2.0	0.17		

Source: Hermann H. Kurzweg, "The Aerodynamic Development of the V-2," in T.H. Benecke and A.W. Quick, eds., ***History of German Guided Missile Development*** (Brunswick, Germany: Verlag E. Appelans, 1957), p. 59, 63. Assumes zero angle of attack; includes jet effects.

Finally, the altitude of the missile is given by

$$h = \sqrt{x^2 + y^2} - R_e \tag{17}$$

where R_e is the radius of the Earth (about 6370 km).

Equation 15 is solved by integrating numerically from $x = 0$ and $y = R_e$ until $z = R_e$, adjusting the initial velocity vector so as to achieve the desired (or maximum) range for a given payload mass.

The Range Equation

The range of a missile depends on its velocity, altitude, and angle at burn-out. The burn-out velocity v_{bo} required for a given range, altitude, and angle is given by[1]

$$v_{bo} = \sqrt{\frac{G\,M_e\,R_e(1-\cos\phi)}{(R_e+h_{bo})^2\sin^2\theta - R_e(R_e+h_{bo})\sin(\theta-\phi)\sin\theta}} \tag{18}$$

where h_{bo} is the burn-out altitude, ϕ is equal to r_b/R_e, where r_b is the ballistic range of the missile, and θ is the angle of the missile at burn-out with respect to the vertical. The maximum range is attained when $\theta = (\phi + \pi)/4$; this is also known as a "minimum-energy" trajectory. The burn-out altitude varies from 30 km for short-range missiles to 200 km to 400 km for ICBMs (see table 2). Table 6 gives v_{bo} for several values of r_b and h_{bo}.

Table 6. The burn-out velocity v_{bo} (km/s) as a function of the maximum ballistic range r_b (km) and burn-out altitude h_{bo} (km).

r_b	Burn-out altitude, h_{bo} (km)					
(km)	0	30	100	200	300	400
500	2.17	2.11	1.97	---	---	---
1,000	3.02	2.97	2.85	---	---	---
2,000	4.11	4.07	3.98	3.86	---	---
3,000	4.87	4.83	4.75	4.64	4.53	4.43
4,000	5.43	5.40	5.32	5.22	5.12	5.02
6,000	6.25	6.22	6.15	6.05	5.96	5.86
8,000	6.81	6.78	6.71	6.61	6.52	6.43
10,000	7.20	7.17	7.10	7.01	6.92	6.83
12,000	7.48	7.45	7.39	7.30	7.21	7.12
14,000	7.68	7.65	7.59	7.50	7.41	7.32

[1] Paul Zarchan, ***Tactical and Strategic Missile Guidance*** (Washington, DC: American Institute of Aeronautics and Astronautics, 1990), p. 232.

Note that r_b includes only the ballistic portion of the trajectory. To estimate the total range r, one must add the downrange distance traveled during the rocket burn: $r = r_b + r_{bo}$. As a rough estimate, $r_{bo} \approx h_{bo} \tan\theta$.

For ranges less than 500 km, the curvature of the Earth can be neglected and equation 18 can be approximated by

$$r_b = \frac{v_{bo}^2 \sin\theta \cos\theta}{g}\left[1 + \sqrt{1 + \frac{2gh_{bo}}{v_{bo}^2 \cos^2\theta}}\right] \tag{19}$$

The maximum range, r_{max}, occurs when $\theta = \pi/4$ (45 degrees), in which case

$$v_{bo} \approx \sqrt{g\left(r_{\max} - 2h_{bo}\right)} \tag{20}$$

where r_{max} includes the distance traveled during boost. The maximum height above the Earth, or apogee, is given by $h_{max} \approx (r_{max}/4)$.

With this background, we can now explore specific missile designs.

China's DF-3 Missile

The Chinese DF-3 missile is a single-stage liquid-fuel intermediate-range ballistic missile (IRBM). The DF-3 booster is also the first stage of the DF-4 ballistic missile and the CZ-1 space launch vehicle (SLV). Because China is marketing space-launch services, it has made information about their SLVs, including the CZ-1, available to the public.

As noted in table 2, the lift-off thrust of the CZ-1 (and therefore of the DF-3) is 104 tonnes (te). The average specific impulse of the engine is 241 seconds—significantly less than the theoretical maximum of 275 s for nitric acid and UDMH (see table 1). The mass of the first stage is 65.5 te, of which 61.4 te is propellant. With this information we can estimate the payload mass or throwweight of the missile as a function of its range. We do this under two assumptions: first, that the propellant mass is a constant 61.4 te; and second, that the propellant mass is increased or decreased to compensate for changes in the payload mass, so that the total mass of the missile remains constant (in this case, 67.5 te).

Constant Propellant Mass. If the propellant mass is held constant at 61.4 te, then the rocket equation gives

$$\frac{M_t - M_p + m_p}{M_t + m_p} = exp\left(- \frac{v_{bo} + \Delta v_{ag}}{v_e} \right) \tag{21}$$

where M_t, M_p, and m_p are the booster, propellant, and payload masses; v_e, the exhaust velocity of the booster, is equal to gI_{sp} = 2.36 km/s. Solving for m_p, we have

$$m_p = \frac{M_t\, exp\left(- \frac{v_{bo} + \Delta v_{ag}}{v_e} \right) - M_t + M_p}{1 - exp\left(- \frac{v_{bo} + \Delta v_{ag}}{v_e} \right)} \tag{22}$$

Table 7 gives the throwweight as a function of the maximum range for h_{bo} = 100 km, r_{bo} = 125 km, and Δv_{ag} = 1.1 km/s. Equation 22 predicts a throwweight of about 2 te at a range of 2800 km, which is in excellent agreement with estimates appearing in the unclassified literature.[2] (Equation 19 gives a burn-out velocity of 4.61 km/s for a maximum range of 2800 km and a burn-out altitude of 100 km.

Constant Total Mass. As an alternative, assume that the total mass of the missile is held constant at a value M_T; in this case the rocket equation gives

$$\frac{M_t - M_p + m_p}{M_T} = exp\left(- \frac{v_{bo} + \Delta v_{ag}}{v_e} \right) \tag{23}$$

which can be solved to give

$$m_p = M_T\, exp\left(- \frac{v_{bo} + \Delta v_{ag}}{v_e} \right) - \left(M_t + M_p \right) \tag{24}$$

The results are shown in table 7, assuming M_T = 67.5 te (i.e., a design throwweight of 2.0 te) and $(M_t - M_p)$ = 4.1 te.[3]

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[2] Mark Wade, "The Chinese Ballistic Missile Program," ***International Defense Review***, August 1990, p. 1191, gives a throwweight of 2 te at a range of 2800 km; the International Institute of Strategic Studies, ***The Military Balance: 1988-1989*** (London: IISS, 1988), p. 219, gives a throwweight of 2 te at a range of 2700 km. John Lewis and Xue Litai, ***China Builds the Bomb*** (Stanford: Stanford University Press, 1988), p. 213, gives a range of 2800 km but no throwweight.

[3] Changes in h_{bo} have little effect on m_p. For example, increasing or decreasing h_{bo} by 20 km increases or decreases m_p by 2.2 percent for constant M_p or 1.9 percent for constant M_T. In the more accurate calculations presented below, calculated burn-out altitudes for optimum trajectories range from 76 to 112 km for constant M_p, and from 92 to 113 km for constant M_T.

Table 7. The throwweight of the DF-3 missile as a function of the maximum range, for a constant propellant mass M_p = 61.4 te; and for a constant total missile mass M_T = 67.5 te.

Range	Throwweight m_p (tonnes)	
(km)	Constant M_p	Constant M_T
1000	11.3	9.6
1500	7.0	6.2
2000	4.4	4.1
2500	2.9	2.8
3000	1.8	1.8
3500	1.0	1.0
4000	0.4	0.5
4500	----	0.02

It should be emphasized that it is not a simple matter to change the throwweight significantly, since this will change the center of mass and therefore the aerodynamic stability of the missile. Deceases in throwweight (and corresponding increases in range) will also lead to higher accelerations and aerodynamic loads during boost, and to higher velocities and increased aerodynamic heating during reentry.

Note that the above analysis, while analytically simple, does not explicitly include the effects of gravity and air resistance on the booster during launch. These effects were included implicitly through Δv_{ag} and h_{bo}. It is instructive to check the accuracy of these calculations by solving numerically the equations of motion, since dependable design information is available for the DF-3. The results, which are given in table 8, are in excellent agreement with those obtained with equations 22 and 24.

Table 8. The maximum range of the DF-3, for a constant propellant mass $M_p = 61.4$ te; and for a constant total missile mass $M_T = 67.5$ te.

Throwweight (te)	Maximum Range r (km) Constant M_p	Constant M_T
0.0	4400	4500
0.5	3800	3900
1.0	3400	3400
2.0	2800	2800
5.0	1800	1700
10.0	1000	900

Israel's Jericho-II/Shavit Missile

Very little is known publicly about the Israeli Jericho II missile. The Shavit space launch vehicle (SLV), which has been used to orbit two Israeli satellites, is widely believed to be based on the Jericho II. From the orbital characteristics and estimated masses of these satellites one can obtain a fairly good idea of the throwweight/range capabilities of the Shavit, and therefore of the Jericho II.

The first satellite, which was launched on 19 September 1988, was placed in an elliptical orbit with a perigee of 250 km, an apogee of 1150 km, and an inclination of 148 degrees; the perigee and apogee of the second satellite were 200 km and 1450 km. (These orbital parameters have been independently verified.) The latitude of the launch site was 32 degrees, and the satellites were launched due west over the Mediterranean Sea (to avoid overflying Arab territory).[4]

The velocity needed to put a satellite into orbit is given by[5]

$$v_c = \sqrt{G\,M_e\left[\frac{2}{R_e} - \frac{1}{a}\right]} \qquad (25)$$

where a, the semi-major axis of the orbit, was 7070 km for the first satellite and 7200 km for the second satellite, which gives $v_c = 8.29$ km/s and 8.35 km/s, respectively.

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[4] Jackson Diehl, "Israel Launches Satellite Into Surveillance Orbit, ***Washington Post***, 4 April 1990, p. A35; Steven E. Gray, Lawrence Livermore National Laboratory, personal communication.

[5] Samual Glasstone, ***Sourcebook on the Space Sciences*** (Princeton: van Nostrand, 1959).

To get the burn-out velocity of the missile, we must add to v_c the component of the earth's rotational velocity in the direction of the launch (Δv_r), as well an amount to compensate for the effects of air resistance and gravity during the rocket burn (Δv_{ag}); Δv_r is given by

$$\Delta v_r = \frac{2\pi R_e}{86164} \cos\Phi \left|\cos\Omega\right| \tag{26}$$

where Φ is the latitude (32 degrees), Ω is the orbital inclination, and 86164 is the number of seconds per sidereal day. Using the orbital inclination for the first satellite (148 degrees), we have $\Delta v_r = 0.33$ km/s.

The Shavit probably has three stages; videotapes of the launches reveal that at least the first stage is solid-fueled. A rough estimate of the Shavit's capbilities can be obtained by assuming that each stage provides the same increase in missile velocity; under these conditions, the mass of the satellite payload would be given by solving equation 5 for the payload mass:

$$m_s \cong M_T \left\{ \left[1 - f\left(1 - exp\left(-\frac{v_c + \Delta v_r + \Delta v_{ag}}{3v_e} \right) \right) \right]^{-3} - 1 \right\}^{-1} \tag{27}$$

where M_T is the mass of the missile (excluding the mass of the payload), f is the ratio of the total mass of each stage to the propellant mass, v_e is the exhaust velocity of each stage.

If the same rocket were used as a ballistic missile, the payload mass for a given burn-out velocity v_{bo} would be given by

$$m_p \cong M_T \left\{ \left[1 - f\left(1 - exp\left(-\frac{v_{bo} + \Delta v_{ag}}{3v_e} \right) \right) \right]^{-3} - 1 \right\}^{-1} \tag{28}$$

The ratio of the ballistic-missile payload mass to the satellite payload mass is therefore given by

$$\frac{m_p}{m_S} \cong \frac{\left[1 - f\left(1 - exp\left(-\frac{v_c + \Delta v_r + \Delta v_{ag}}{3v_e} \right) \right) \right]^{-3} - 1}{\left[1 - f\left(1 - exp\left(-\frac{v_{bo} + \Delta v_{ag}}{3v_e} \right) \right) \right]^{-3} - 1} \tag{29}$$

All that remains is to substitute values for f, v_e, and Δv_{ag} into equation 24, along with the estimates of v_c and Δv_r derived above. For a large, solid-fuel rocket, typical values are $f = 1.15$, $v_e = 2.6$ km/s, and $\Delta v_{ag} = 1.0$ km/s.[6]

The satellite mass given by Israeli was 156 kg for the first satellite and 170 kg for the second satellite; including a guidance and control package would bring the total satellite payload mass to at least 200 kg. Assuming that $m_s = 200$ kg, the throwweight predicted by equation 24 is given in table 9 as a function of the maximum range of the missile, assuming a burn-out altitude and range of 200 and 450 km, respectively. Some analysts have speculated that the Jericho II is simply is the first two stages of the Shavit SLV; table 9 also gives the throwweight of the first two stages of a three-stage Shavit.

Table 9. The throwweight of the Shavit SLV as a function of the maximum range, and the throwweight of the first two stages.

Range (km)	Throwweight m_p (tonnes) First 2 stages	Throwweight m_p (tonnes) All 3 stages
1000	8.3	11.7
1500	5.2	6.9
2000	3.7	4.8
3000	2.2	2.7
4000	1.4	2.0
5000	1.0	1.5
6000	0.8	1.2
8000	0.5	0.8
10000	0.3	0.6

Another—and better—way to estimate the capability of the Israeli missile is to compare it to a missile of similar size whose details are well known, scaling the mass up or down to achieve the orbital capability demonstrated by the Shavit. The total missile mass given by equation 22 is 33 te, which is about the same as the U.S. Minuteman-II missile (32.5 te). Both missiles use solid propellants, and it is reasonable to assume that Israel could achieve the same level of performance as that of the 1960s-vintage Minuteman II.

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[6] See table 2. Also see Glasstone, ***Sourcebook on the Space Sciences***, and Sharkey, "The Rocket Performance Computer."

Using equations 2 and 3 and the missile parameters given in table 2, one finds that the Minuteman II could deliver a 700-kg payload to a range of 10,000 km, which is in good agreement with the 800-kg throwweight declared by the United States under the START Treaty.[7] Using the same assumptions, the Minuteman-II missile would be capable of launching a 160-kg satellite into the same orbit as the first Israeli satellite. Thus, the Shavit and the Minuteman II are missiles of similar capability. (The Minuteman III, with its more advanced third stage, is capable of launching 300-kg satellite into the same orbit.)

Table 10 gives the throwweight of the Minuteman II as a function of the maximum range, assuming constant propellant mass. The throwweight of the Shavit, if used as a ballistic missile, should be similar. Table 10 also gives the throwweight of the first two stages of the Minuteman II; if the Jericho II is indeed the first two stages of the Shavit, then the throwweight of the first two Minuteman stages should be comparable to that of the Jericho II. Despite the simplicity of the earlier estimates based on equation 24, the correspondence between tables 9 and 10 is remarkably good.

It is interesting to compare the throwweight of the first two stages of the Minuteman II with that of the DF-3. At a range of 1700 km both missiles have roughly equal throwweights (about 5 te). At longer ranges, however, the two-stage missile has the advantage: although the DF-3 is limited to ranges of less than 4000 km, the first two stages of the Minuteman could deliver a 200-kg payload at intercontinental ranges. This clearly demonstrates the importance of multi-stage rocket technology for long-range delivery.

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[7] "Memorandum of Understanding on the Establishment of the Data Base Relating to the Treaty Between the United States of America and the Union of Soviet Socialist Republics on the Reduction and Limitation of Strategic Offensive Arms," dated 1 September 1990, gives a throwweight of 800 kg for the Minuteman II. According to the Treaty, this may be the greatest throwweight demonstrated in flight tests (excluding the first seven tests, unless the throwweight in one of the these tests exceeds by more than 20 percent the throwweight in subsequent tests), or the throwweight at 11,000 km, whichever is greater. Using the equations presented here, Minuteman II would have a throwweight of 800 kg at a range of 9,000 km.

Table 10. The throwweight of the Minuteman II missile as a function of the maximum range, and the throwweight of the first two stages.

Range (km)	Throwweight m_p (tonnes) First 2 stages	All 3 stages
1000	8.4	12.9
1500	5.5	7.6
2000	3.9	5.3
3000	2.4	3.2
4000	1.6	2.2
5000	1.1	1.7
6000	0.8	1.3
8000	0.4	0.9
10000	0.2	0.7

The Soviet Scud-B Missile

The Scud-B is a single-stage, liquid-fuel, short-range ballistic missile based on German V-2 rocket technology. Like the Chinese DF-3 (which itself is based on early Soviet technology), the propellants are UDMH and IRFNA, so we may assume about the same exhaust velocity (2.4 km/s) as the DF-3. The missile is widely attributed with a throwweight of 1 te at a maximum range of 300 km.[8] Zaloga gives a total missile mass of 5.9 te and a propellant mass of 3.7 te;[9] assuming a 1.0-te payload, the ratio of total booster mass to propellant mass f would be 1.32. Although this ratio is large by modern standards, the V-2 missile had about the same ratio.[10] We can also assume that the Scud has about the same burn-out altitude as the V-2 (28 km) and the same Δv_{ag} (about 0.7 km/s).

With these booster characteristics we can use equations 17 and 19 to estimate the Scud's throwweight at a given range. Very similar results are obtained if we assume that the Scud has the same initial thrust-to-weight ratio as the V-2 (i.e., a thrust of 12 te) and use equation 10 to estimate the maximum range for a given

[8] Steven Zaloga, "Ballistic Missiles in the Third World: SCUD and Beyond," ***International Defense Review***, November 1988, pp. 1423-1427; Thomas B. Cochran, William M. Arkin, Robert S. Norris, and Jeffrey I. Sands, ***Nuclear Weapons Databook, Vol. IV: Soviet Nuclear Weapons*** (New York: Ballinger, 1989), p. 220-222.
[9] Zaloga, "Ballistic Missiles," p. 1427.
[10] Gregory P. Kennedy, ***Vengeance Weapon 2: The V-2 Guided Missile*** (Washington DC: Smithsonian Institution Press, 1983), p. 84.

payload; these results appear in table 11. The assumptions about the missile's characteristics appear to be accurate, inasmuch as the throwweight is predicted to be 1.0 te at 300 km.

Iraq claimed that by reducing the throwweight they were able to extend the Scud's range to 650 km; table 11 shows that the throwweight at this range would be only one-eighth of its throwweight at 300 km if the propellant mass remained constant. If, on the other hand, propellant is added to compensate for the reduced payload mass (i.e., a constant total missile mass of 5.9 te), then the throwweight would be about 300 kg at a range of 650 km. Since Iran claimed that the modified Scud (dubbed the *"al Hussein"* by Iraq) carried a warhead weighing 160 to 180 kg, constant propellant mass is the best assumption.[11] Iraq fired nearly 200 *al Husseins* against Iranian cities; many were launched at Teheran, which was over 500 km from the nearest Iraqi launch sites. A throwweight of 200 kg is consistent with available estimates of the damage caused by these attacks.[12]

It was widely reported that Iraq had further extended the range of the Scud-B by lengthening the missile to carry additional propellant; apparently two *al Abbas* missiles were made from three cannibalized Scuds. This can result in a increase in throwweight much greater than 50 percent, since the mass of the engines and tail section stay the same—only the mass of the fuel tanks must increase. In the V-2, for example, the fuel tanks account for only one-quarter of the mass of the empty booster. If we assume the same fraction for the Scud-B, and further assume that the *al Abbas* modification results in a 50% increase in the fuel-tank and propellant mass only, then $f = 1.25$ for the modified missile. The throwweight as a function of range under these assumptions is given in table 11. Note that the *al Abbas* could deliver a 1-te payload to a range of 440 km, and that its throwweight at 900 km would equal that of the Scud-B at only 650 km (assuming a constant propellant mass). These estimates are consistent with reports that the *al Abbas* had a range of up to 900 km, and that it could deliver the normal 1.0-te Scud warhead to ranges significantly greater than the *al Hussein* missile could.[13]

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[11] W. Seth Carus and Joseph S. Bermudez, Jr, "Iraq's *Al-Husayn* Missile Programme," ***Jane's Soviet Intelligence Review***, May 1990, pp. 205, states that Iran claims that the *al Hussein* missile had a payload containing 190 kg of explosives and a maximum range of 600 km. They also state that the fuel tanks were lengthened and 1040 kg of additional propellant added to the missile, but this clearly would not be necessary to achieve such a small throwweight at this range.
[12] See appendix B [NEED NEW FOOTNOTE HERE].
[13] See Aaron Karp, "Ballistic Missile Proliferation," p. 386; Zagola, "Ballistic Missiles," p. 1425.

<u>Table 11</u>. The range of the Scud-B and *al Abbas* missiles as a function of throwweight, for constant propellant mass (3.7 te for Scud, 5.55 te for *al Abbas*); and constant missile mass (5.9 te for Scud, 7.9 te for *al Abbas*).

	Maximum Range r (km)			
Throwweight	Scud-B / *al Hussein*		*al Abbas*	
(tonnes)	Constant M_p	Constant M_T	Constant M_p	Constant M_T
0.0	730	980	1000	1150
0.125	640	840	890	1010
0.25	560	720	800	890
0.5	450	530	640	700
1.0	300	300	440	440

<u>Missile Ranges in Perspective</u>

To put these and other ranges given in this appendix in perspective, table 12 gives the minimum range between several countries that possess ballistic missiles and major cities in the Middle East region. Note that every city listed (and many major cities not listed) is within IRBM range of every emerging missile-capable country. For example, every city is within the range of the DF-3 missile (2,800 km) from Saudi Arabia and Iraq, and every city but Tripoli is within DF-3 range of Iran. The Middle East is a small neighborhood; even missiles with ranges of 1,000 km or less can strike many potential adversaries.

Table 12. Minimum range (km) from several countries with ballistic missile programs to major cities in the region.

	Missile Launched From					
Target City	Egypt	India	Iran	Iraq	Israel	Saudi Arabia
Cairo, Egypt	——	3600	1400	800	300	400
Delhi, India	4100	——	1400	2800	4000	2300
Teheran, Iran	1600	1900	——	400	1400	800
Baghdad, Iraq	1000	2400	120	——	800	300
Tel Aviv, Israel	100	3300	1000	400	——	250
Tripoli, Libya	1200	5400	3100	2500	2100	2200
Karachi, Pakistan	3100	170	600	1900	3100	1200
Riyadh, Saudi Arabia	1100	2200	400	500	1300	——
Damascus, Syria	300	3100	800	230	60	240
Ankara, Turkey	900	3500	900	800	800	1000
Baku, Azerb.	1600	2200	170	600	1400	1200
Sanaa, Yemem	800	2700	1000	1000	1300	200

Cost-effectiveness of Missiles vs. Aircraft

Why use missiles instead of aircraft? After all, aircraft are reusable, they are capable of much better accuracy than first-generation missiles, and it is much simpler to deliver many payloads (e.g. chemical agents) with aircraft than missiles. The usual answer is that the higher velocity of missiles gives them a much better chance of penetrating to their targets. Moreover, missiles do not require highly skilled pilots and better control can be maintained over missiles than aircraft (missiles cannot defect). A detailed answer must take into account the relative costs of missiles and aircraft for a given payload and range, as well as the relative probability that they will penetrate to their targets.

Table 13 gives the mass and unit flyaway cost of several U.S. solid-fuel ballistic missiles in 1986 dollars. Note that the price per kilogram varies by only a factor of four, even though the missile mass varies by a factor 70. The average cost is about \$200/kg, plus or minus a factor of two. If we apply this cost per unit mass to the liquid-fuel Scud and DF-3 (admittedly a questionable procedure), we find that they cost roughly \$1 million and \$10 million each, respectively.

Table 13. The mass (in tonnes) and unit flyaway cost (in million FY 1986 dollars) of several U.S. ballistic missiles.

Missile	Mass (te)	Unit Flyaway Cost (million FY 1986$)	Cost/Mass ($/kg)
Minuteman III	35	7.8	220
MX/Peacekeeper	88	22	250
Poseidon C3	29	5.0	170
Trident C4	30	8.1	270
Trident D5	57	28	490
Lance	1.3	0.16	120
Pershing II	7.4	2.5	340

Source: Thomas B. Cochran, William M. Arkin, and Milton M. Hoenig, ***Nuclear Weapons Databook, Vol. I: U.S. Nuclear Forces and Capabilities*** (Cambridge, MA: Ballinger, 1984).

For comparison, table 14 gives the mass, payload, combat radius, and unit flyaway cost of several U.S. aircraft. Once again, the flyaway cost per unit takeoff mass varies by only a factor of four from the least expensive aircraft (A-7) to the most expensive (B-2). The average cost for fighter and attack aircraft is roughly $700 per kilogram of takeoff mass.

Referring to table 14, the U.S. A-7 aircraft has a maximum payload of 5.9 te, a combat radius of 880 km, and a unit flyaway cost of $8.5 million dollars. To deliver an equal payload at a range of only 300 km would require at six Scud-B missiles at a cost of roughly $6 million dollars. Given the unreliability and inaccuracy of ballistic missiles, an A-7 would only have to complete an average of one mission (an attrition rate of 50%) to be cost effective compared the Scud, other considerations aside.

As another example, consider the comparison between the U.S. A-6 and the Chinese DF-3 missile. Both delivery systems have (or could have) roughly equal payload/range capabilities; the A-6 costs about $19 million, while the DF-3 costs about $10 million. Once again, attrition rates must be very high to make the DF-3 a cost-effective alternative. Since most air defenses cannot impose such high attrition rates, the popularity of ballistic missiles must be explained by other factors, such as speed, political control, prestige, or psychological impact.

Table 14. The maximum takeoff weight, payload, combat radius, and unit flyaway cost in 1986 dollars of several U.S. aircraft.

Aircraft	Takeoff Mass (te)	Payload (te)	Combat Radius (km)	Flyaway Cost (million 1986$)	Cost/Mass ($/kg)
A-4	11.6	4.5	1250	7.7	660
A-6	27.4	8.2	1250	19	690
A-7	19	5.9	880	8.5	450
F-15	31	7.3	1350	22	710
F-16	15	5.4	930	13	870
F-18	20	7.7	850	24	1200
B-1	217	29	4600	228	1000
B-2	168	23	5000	274	1600

Source: Thomas B. Cochran, William M. Arkin, and Milton M. Hoenig, ***Nuclear Weapons Databook, Vol. I: U.S. Nuclear Forces and Capabilities*** (Cambridge, MA: Ballinger, 1984), and International Institute for Strategic Studies, ***The Military Balance 1988-1989*** (London: IISS, 1988).